Nonfactorizable contributions to B meson decays into charmonia

Chuan-Hung Chen[1*] **and Hsiang-nan Li**[1,2†]

[1] *Department of Physics, National Cheng-Kung University, Tainan, Taiwan 701, Republic of China*

[2] *Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China*

Abstract

We analyze the $B \to (J/\psi, \chi_{c0}, \chi_{c1}, \eta_c)K^{(*)}$ decays, which have small or even vanishing branching ratios in naive factorization assumption (FA). We calculate nonfactorizable corrections to FA in the perturbative QCD approach based on k_T factorization theorem. The charmonium distribution amplitudes are inferred from the non-relativistic heavy quarkonium wave functions. It is found that the nonfactorizable contributions enhance the branching ratios and generate the relative phases among helicity amplitudes of the above modes. Most of the observed branching ratios, polarization fractions, and relative phases, except those of $B \to \eta_c K$, are explained. Our predictions for the $B \to (\chi_{c0}, \chi_{c1}, \eta_c)K^*$ decays can be compared with future data.

*chchen@mail.ncku.edu.tw

†hnli@phys.sinica.edu.tw

1 INTRODUCTION

It has been known that the naive factorization assumption (FA) [1, 2] does not apply to exclusive B meson decays into charmonia, such as $B \to J/\psi K$ [3]. These modes belong to the color-suppressed category [4], for which predictions from FA are always small due to the vanishing Wilson coefficient $a_2 \sim 0$. However, the branching ratios measured by Babar recently [5]

$$\begin{aligned} B(B^+ \to J/\psi K^+) &= (10.61 \pm 0.15 \pm 0.48) \times 10^{-4} , \\ B(B^0 \to J/\psi K^0) &= (8.69 \pm 0.22 \pm 0.30) \times 10^{-4} , \end{aligned} \tag{1}$$

imply a larger parameter $a_2(J/\psi K) \approx 0.20 - 0.30$ [3]. The same difficulty has appeared in other similar decays $B \to (\chi_{c0}, \chi_{c1}, \eta_c)K$: the observed branching ratios are usually many times larger than the expectations from FA. For example, the $B \to \chi_{c0}K$ decays do not receive factorizable contributions, but the data of their branching ratios are comparable to those of $B \to J/\psi K$ in Eq. (1).

Many attempts to resolve this puzzle have been made in more sophisticated approaches (for a review, see [6]). The large $a_2(J/\psi K)$ leads to a natural conjecture that nonfactorizable contributions, such as the vertex and spectator corrections from Figs. 1(a)-1(d) and from Figs. 1(e) and 1(f), respectively, may play an important role. It has been found [7] in the QCD-improved factorization (QCDF) [8] that the above nonfactorizable contributions, resulting in the branching ratio $B(B^0 \to J/\psi K^0) \approx 1 \times 10^{-4}$, are too small to explain the data. Since only the leading-twist (twist-2) kaon distribution amplitude was included in [7], the authors of [9] added the twist-3 contribution, which is chirally enhanced, though being power-suppressed in the heavy quark limit. Unfortunately, Figs. 1(e) and 1(f) generate logarithmical divergences from the end-point region, where the parton momentum fraction of the kaon is small. To make an estimation, arbitrary cutoffs for parameterizing the divergences, i.e., large theoretical uncertainties, have been introduced. The end-point singularities become more serious in the QCDF analysis of the $B \to (\chi_{c0}, \chi_{c1})K$ decays [10]. It has been claimed that the data of those modes involving J/ψ, χ_{c1}, and η_c are not understandable in QCDF, and only those involving χ_{c0} are [7, 10, 11].

The $B \to J/\psi K$ decays have been also studied in light-cone sum rules (LCSR) [12, 13], and the small branching ratio from the factorizable contribution [12],

$$B(B \to J/\psi K)_{\rm fact} \approx 3 \times 10^{-4} , \tag{2}$$

was confirmed. The nonfactorizable contributions considered in [12, 13] refer, however, to those arising from the three-parton kaon distribution amplitudes. In the corresponding diagrams an additional valence gluon from the kaon attaches one of the charm quarks in the J/ψ meson. Adding these pieces, one arrived at the parameter $a_2(J/\psi K) \sim 0.14 - 0.17$, which is still insufficient to account for the data. The calculation of Fig. 1 in LCSR, involving two-loop integrals, has not yet been performed. Hence, the conclusion in [12, 13] simply indicates that the nonfactorizable contributions to the $B \to J/\psi K$ decays from higher Fock states of the involved mesons are negligible.

We doubt that the conclusion from QCDF is solid for two reasons. First, the end-point singularities render the estimation of the nonfactorizable contributions out of control. Second, the simple asymptotic models were adopted for the charmonium distribution amplitudes, which,

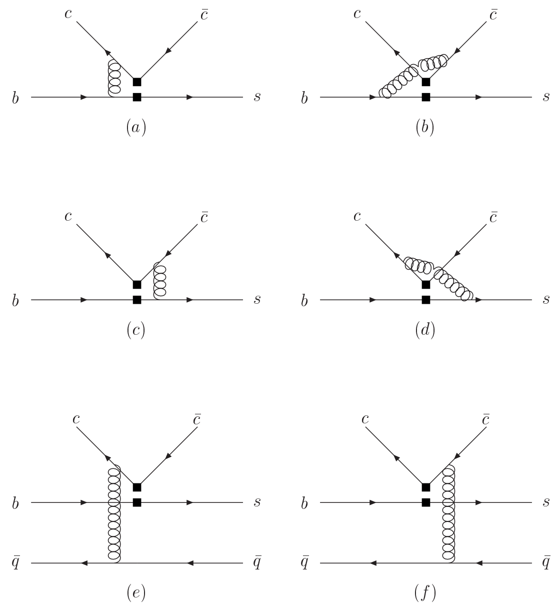

Figure 1: Feynman diagrams for nonfactorizable corrections to $\overline{B} \to J/\psi K$.

without any theoretical and experimental bases, may underestimate the nonfactorizable contributions. Motivated by the above observations, we shall investigate the nonfactorizable corrections to FA from Fig. 1 using the following method. We take into account the vertex corrections through the variation of the renormalization scale for the factorizable contributions, because their evaluation requires more theoretical inputs, which have not yet been available. The spectator amplitudes will be computed in the perturbative QCD (PQCD) approach based on k_T factorization theorem [14, 15, 16, 17]. The formalism, similar to that of the color-suppressed mode $\bar{B}^0 \to D^0\pi^0$ [18], is free from the end-point singularities. We infer the charmonium distribution amplitudes from the non-relativistic heavy quarkonium bound-state wave functions, which have been shown to well explain the cross section of charmonium production in e^+e^- collisions [19]. Based on the universality of hadron distribution amplitudes, these more sophisticated models should be able to account for the exclusive B meson decays into charmonia, if QCD factorization theorem works.

The $B \to J/\psi K^{(*)}$ decays have been studied in PQCD [20, 21, 22], compared to which the new ingredients of this work are: 1) we clarify several controversial statements on the end-point singularities in the $B \to (J/\psi, \chi_{c1})K$ analysis in the literature; 2) the $K^{(*)}$ meson distribution amplitudes derived from QCD sum rules [23, 24] are included up to two-parton and twist-3 level; 3) the models for the charmonium distribution amplitudes have both theoretical and experimental bases; 4) not only the $B \to J/\psi K^{(*)}$ decays, but $B \to (\chi_{c0}, \chi_{c1}, \eta_c)K^{(*)}$ are investigated. It will be shown that the obtained branching ratios, polarization fractions, and relative phases among various helicity amplitudes, except those of $B \to \eta_c K$, are all in consistency with the existing data. The observed $B \to \eta_c K$ branching ratios, significantly larger than our results, are the only puzzle. Therefore, our conclusion differs from that drawn

in QCDF [7, 10, 11]. The predictions for the $B \to (\chi_{c0}, \chi_{c1}, \eta_c)K^*$ modes can be compared with future measurements.

We present our formalism for the $B \to J/\psi K^{(*)}$ decays as an example in Sec. II. The factorizable contributions are treated in FA, since the $B \to K^{(*)}$ transition form factors in the present case are characterized by a low scale, and may not be calculable. The vertex and spectator corrections are handled in the way stated above. The same formalism is then applied to the other exclusive B meson decays into charmonia in Sec. III. Section IV is the conclusion. The Appendix collects the expressions of the involved meson distribution amplitudes and of the factorization formulas for all the spectator amplitudes. For a review of the studies of semi-inclusive B meson decays into charmonia, refer to [25].

2 THE $B \to J/\psi K^{(*)}$ DECAYS

We write the B (J/ψ) meson momentum P_1 (P_2) in the light-cone coordinates as

$$P_1 = \frac{m_B}{\sqrt{2}}(1, 1, \mathbf{0}_T) \;, \quad P_2 = \frac{m_B}{\sqrt{2}}\left(1, r_2^2, \mathbf{0}_T\right) \;, \tag{3}$$

with the mass ratio $r_2 = m_{J/\psi}/m_B$. The $K^{(*)}$ meson momentum is then given by $P_3 = P_1 - P_2$. The polarization vectors of the J/ψ meson are parameterized as

$$\epsilon_{2L} = \frac{1}{\sqrt{2}r_2}\left(1, -r_2^2, \mathbf{0}_T\right) \;, \quad \epsilon_{2T} = (0, 0, \mathbf{1}_T) \;. \tag{4}$$

The relevant effective Hamiltonian for the $B \to J/\psi K^{(*)}$ decays is

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\left\{V_{cb}V_{cs}^*[C_1(\mu)O_1 + C_2(\mu)O_2] - V_{tb}V_{ts}^*\sum_{k=3}^{10} C_k(\mu)O_k\right\} \;, \tag{5}$$

with the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V and the four-fermion operators,

$$\begin{aligned}
&O_1 = (\bar{s}_i c_j)_{V-A}(\bar{c}_j b_i)_{V-A} \;, &&O_2 = (\bar{s}_i c_i)_{V-A}(\bar{c}_j b_j)_{V-A} \;,\\
&O_3 = (\bar{s}_i b_i)_{V-A}\sum_q(\bar{q}_j q_j)_{V-A} \;, &&O_4 = (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A} \;,\\
&O_5 = (\bar{s}_i b_i)_{V-A}\sum_q(\bar{q}_j q_j)_{V+A} \;, &&O_6 = (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A} \;,\\
&O_7 = \frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V+A} \;, &&O_8 = \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V+A} \;,\\
&O_9 = \frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V-A} \;, &&O_{10} = \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V-A} \;,
\end{aligned} \tag{6}$$

i, j being the color indices. Below we shall neglect the tiny product $V_{ub}V_{us}^*$, and assume $V_{cb}V_{cs}^* = -V_{tb}V_{ts}^*$ for convenience.

2.1 Factorization Formulas

The $B \to J/\psi K$ decay rate has the expression,

$$\Gamma = \frac{1}{32\pi} G_F^2 |V_{cb}|^2 |V_{cs}|^2 m_B^3 (1-r_2^2)^3 |\mathcal{A}|^2 \ . \tag{7}$$

Computing the factorizable contribution to the amplitude $\mathcal{A}$ in PQCD, we found that its characteristic scale is around 1 GeV. With such a small hard scale, the perturbation theory may not be reliable. Therefore, we do not attempt to calculate the factorizable contribution, but parameterize it in FA. The amplitude $\mathcal{A}$ is then written as

$$\mathcal{A} \;=\; a_{\rm eff}(\mu) f_{J/\psi} F_1(m_{J/\psi}^2) + \mathcal{M}^{(J/\psi K)} \ , \tag{8}$$

with the J/ψ meson decay constant $f_{J/\psi}$ and the spectator amplitude $\mathcal{M}^{(J/\psi K)}$. The form factor $F_1(q^2)$ is defined via the matrix element,

$$\langle K(P_3)|\bar{b}\gamma_\mu s|B(P_1)\rangle = F_1(q^2)\left[(P_1+P_3)_\mu - \frac{m_B^2 - m_K^2}{q^2} q_\mu\right] + F_0(q^2)\frac{m_B^2 - m_K^2}{q^2} q_\mu \ , \tag{9}$$

$q = P_1 - P_3$ being the momentum transfer, and m_K the kaon mass. The effective Wilson coefficient $a_{\rm eff}$ sums the contributions from both the tree and penguin operators in Eq. (5):

$$a_{\rm eff}(\mu) = a_2(\mu) + a_3(\mu) + a_5(\mu) \ , \tag{10}$$

with

$$\begin{aligned} a_2 &= C_1 + \frac{C_2}{N_c} \ , \\ a_3 &= C_3 + \frac{C_4}{N_c} + \frac{3}{2} e_c \left(C_9 + \frac{C_{10}}{N_c}\right) \ , \\ a_5 &= C_5 + \frac{C_6}{N_c} + \frac{3}{2} e_c \left(C_7 + \frac{C_8}{N_c}\right) \ , \end{aligned} \tag{11}$$

and the charm quark charge $e_c = 2/3$.

After parameterizing the factorizable contribution in FA, the soft dynamics is absorbed into the form factor F_1. The remaining piece, i.e., the Wilson coefficient, is dominated by hard dynamics of $O(m_b)$, m_b being the b quark mass. We shall set the renormalization scale to $\mu = m_b \approx 4.4$ GeV, which corresponds to $a_{\rm eff}(m_b) \approx 0.1$. The spectator amplitude $\mathcal{M}^{(J/\psi)}$ is characterized by a scale of $O(\sqrt{\bar{\Lambda} m_b})$ with $\bar{\Lambda}$ being a hadronic scale [18, 26, 27]: the hard gluons in Figs. 1(e) and 1(f), carrying the difference between the momenta of the soft spectator in the B meson and of the energetic spectator in the kaon, are off-shell by $O(\sqrt{\bar{\Lambda} m_b})$. Strictly speaking, once a B meson transition form factor is treated as a soft object, the vertex corrections in Figs. 1(a)-1(d) should be added, which become of the same order as the spectator amplitude. This is the counting rules of QCDF. The PQCD formalism for the vertex corrections requires the inclusion of the transverse momenta k_T of the charm quarks, because of the end-point singularities in some modes. The k_T-dependent charmonium wave functions are then the necessary inputs, which, however, have been determined neither theoretically nor experimentally.

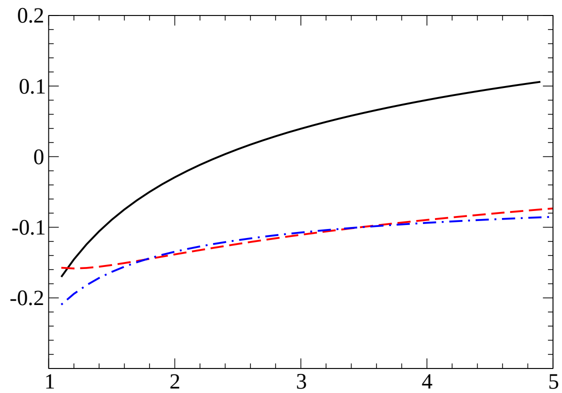

Figure 2: Dependence of $a_{\rm eff}$ on the renormalization scale μ. The unit for the horizontal axis is GeV. The solid, dashed and dash-dotted lines stand for $a_{\rm eff}$ without the vertex corrections, the real part of $a_{\rm eff}$ with the vertex corrections, and the imaginary part of $a_{\rm eff}$ with the vertex corrections, respectively.

Therefore, the vertex corrections appear as a theoretical uncertainty eventually, which can be covered by a variation of the scale μ between $0.5m_b$ and $1.5m_b$ as shown below.

In terms of the notation in [28], we decompose the nonlocal matrix elements associated with longitudinally and transversely polarized J/ψ mesons into

$$\langle J/\psi(P,\epsilon_L)|\bar{c}(z)_j c(0)_l|0\rangle = \frac{1}{\sqrt{2N_c}}\int_0^1 dx e^{ixP\cdot z}\Big\{m_{J/\psi}[\not\epsilon_L]_{lj}\Psi^L(x)+[\not\epsilon_L\ \not P]_{lj}\Psi^t(x)\Big\}\ , \quad (12)$$

$$\langle J/\psi(P,\epsilon_T)|\bar{c}(z)_j c(0)_l|0\rangle = \frac{1}{\sqrt{2N_c}}\int_0^1 dx e^{ixP\cdot z}\Big\{m_{J/\psi}[\not\epsilon_T]_{lj}\Psi^V(x)+[\not\epsilon_T\ \not P]_{lj}\Psi^T(x)\Big\}\ , \quad (13)$$

respectively, which define the twist-2 distribution amplitudes Ψ^L and Ψ^T, and the twist-3 distribution amplitudes Ψ^t and Ψ^V with the c quark carrying the fractional momentum xP. It is confusing that both the structures $\not\epsilon_L$ and $\not\epsilon_L\ \not P$ were regarded as being leading-twist in the analysis of the $B\to J/\psi K$ decays [7, 9]. In fact, it is $\not\epsilon_T\ \not P$ that is leading-twist, which contributes only to the $B\to J/\psi K^*$ decays.

The vertex corrections to the $B\to J/\psi K$ decays, denoted as f_I in QCDF, have been calculated in the NDR scheme [7, 9]. Their effect can be combined into the Wilson coefficients associated with the factorizable contributions:

$$\begin{aligned} a_2 &= C_1+\frac{C_2}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_2\left(-18+12\ln\frac{m_b}{\mu}+f_I\right)\ , \\ a_3 &= C_3+\frac{C_4}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_4\left(-18+12\ln\frac{m_b}{\mu}+f_I\right)\ , \\ a_5 &= C_5+\frac{C_6}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_6\left(6-12\ln\frac{m_b}{\mu}-f_I\right)\ , \end{aligned} \quad (14)$$

with the function,

$$f_I=\frac{2\sqrt{2N_c}}{f_{J/\psi}}\int dx_3\Psi^L(x_2)\left[\frac{3(1-2x_2)}{1-x_2}\ln x_2-3\pi i+3\ln(1-r_2^2)+\frac{2r_2^2(1-x_2)}{1-r_2^2x_2}\right]\ , \quad (15)$$

where those terms proportional to r_2^4 have been neglected. We have also neglected the contributions from the electroweak penguin operators for simplicity. As shown in Fig. 2, including the vertex corrections leads to a larger $|a_{\rm eff}(m_b)| \approx 0.12$, which reproduces Eq. (2) roughly. We have confirmed that the variation of μ between $0.5m_b$ and $1.5m_b$ is wide enough for taking into account the effect of the vertex corrections.

The definitions of the B meson wave function Φ_B and of the kaon distribution amplitudes Φ_K are referred to [27, 28]. It is then straightforward to derive the spectator amplitude,

$$\mathcal{M}^{(J/\psi K)} = \mathcal{M}_4^{(J/\psi K)} + \mathcal{M}_6^{(J/\psi K)} , \tag{16}$$

where the amplitudes $\mathcal{M}_4^{(J/\psi K)}$ and $\mathcal{M}_6^{(J/\psi K)}$ result from the $(V-A)(V-A)$ and $(V-A)(V+A)$ operators in Eq. (5), respectively. Their factorization formulas are given by

$$\begin{aligned}
\mathcal{M}_4^{(J/\psi K)} &= 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1)\\
&\times\Big\{\Big[(1-2r_2^2)(1-x_2)\Phi_K(x_3)\Psi^L(x_2)+\frac{1}{2}r_2^2\Phi_K(x_3)\Psi^t(x_2)\\
&-r_K(1-r_2^2)x_3\Phi_K^p(x_3)\Psi_L(x_2)+r_K\left(2r_2^2(1-x_2)+(1-r_2^2)x_3\right)\Phi_K^\sigma(x_3)\Psi^L(x_2)\Big]\\
&\times E_4(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1)\\
&-\Big[(x_2+(1-2r_2^2)x_3)\Phi_K(x_3)\Psi^L(x_2)+r_2^2(2r_K\Phi_K^\sigma(x_3)-\frac{1}{2}\Phi_K(x_3))\Psi^t(x_2)\\
&-r_K(1-r_2^2)x_3\Phi_K^p(x_3)\Psi_L(x_2)-r_K\left(2r_2^2x_2+(1-r_2^2)x_3\right)\Phi_K^\sigma(x_3)\Psi^L(x_2)\Big]\\
&\times E_4(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1) ,
\end{aligned}\tag{17}$$

$$\begin{aligned}
\mathcal{M}_6^{(J/\psi K)} &= 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1)\\
&\times\Big\{\Big[(1-x_2+(1-2r_2^2)x_3)\Phi_K(x_3)\Psi^L(x_2)+r_2^2(2r_K\Phi_K^\sigma(x_3)-\frac{1}{2}\Phi_K(x_3))\Psi^t(x_2)\\
&-r_K(1-r_2^2)x_3\Phi_K^p(x_3)\Psi^L(x_2)-r_K\left(2r_2^2(1-x_2)+(1-r_2^2)x_3\right)\Phi_K^\sigma(x_3)\Psi^L(x_2)\Big]\\
&\times E_6(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1)\\
&-\Big[(1-2r_2^2)x_2\Phi_K(x_3)\Psi^L(x_2)+\frac{1}{2}r_2^2\Phi_K(x_3)\Psi^t(x_2)\\
&-r_K(1-r_2^2)x_3\Phi_K^p(x_3)\Psi^L(x_2)+r_K\left(2r_2^2x_2+(1-r_2^2)x_3\right)\Phi_K^\sigma(x_3)\Psi^L(x_2)\Big]\\
&\times E_6(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\} ,
\end{aligned}\tag{18}$$

with the color factor $C_F = 4/3$, the number of colors $N_c = 3$, the symbol $[dx] \equiv dx_1dx_2dx_3$ and the mass ratio $r_K = m_0^K/m_B$, m_0^K being the chiral scale associated with the kaon.

The evolution factors are written as

$$E_i(t) = \alpha_s(t)a_i'(t)S(t)|_{b_3=b_1} , \tag{19}$$

with the Wilson coefficients,

$$\begin{aligned}
a_4' &= \frac{C_2}{N_c}+\frac{1}{N_c}\left(C_4+\frac{3}{2}e_cC_{10}\right) ,\\
a_6' &= \frac{1}{N_c}\left(C_6+\frac{3}{2}e_cC_8\right) .
\end{aligned}\tag{20}$$

The Sudakov exponent is given by

$$
\begin{aligned}
S(t) &= S_B(t)+S_K(t)\ ,\\
S_B(t) &= \exp\left[-s(x_1P_1^+,b_1)-\frac{5}{3}\int_{1/b_1}^t\frac{d\bar\mu}{\bar\mu}\gamma(\alpha_s(\bar\mu))\right]\ , &(21)\\
S_K(t) &= \exp\left[-s(x_3P_3^-,b_3)-s((1-x_3)P_3^-,b_3)-2\int_{1/b_3}^t\frac{d\bar\mu}{\bar\mu}\gamma(\alpha_s(\bar\mu))\right]\ , &(22)
\end{aligned}
$$

with the quark anomalous dimension $\gamma=-\alpha_s/\pi$. Note that the coefficient 5/3 of the quark anomalous dimension in Eq. (21) differs from 2 in Eq. (22). The reason is that the rescaled heavy-quark field adopted in the definition of the heavy-meson wave function has a self-energy correction different from that of the full heavy-quark field [29]. The variables b_1 and b_3, conjugate to the parton transverse momenta k_{1T} and k_{3T}, represent the transverse extents of the B and K mesons, respectively. The explicit expression of the exponent s can be found in [30, 31, 32]. The above Sudakov exponentials decrease fast in the large b region, such that the $B\to J/\psi K$ spectator amplitudes remain sufficiently perturbative in the end-point region of the momentum fractions.

The hard functions $h_d^{(j)}$, $j=1$ and 2, are

$$
h_d^{(j)} = \frac{1}{D-D_j}\begin{pmatrix} K_0(\sqrt{D_j}m_Bb_1)-K_0(\sqrt{D}m_Bb_1) & \text{for } D_j\geq 0\\ \frac{i\pi}{2}H_0^{(1)}\left(\sqrt{|D_j|}m_Bb_1\right)-K_0(\sqrt{D}m_Bb_1) & \text{for } D_j<0\end{pmatrix}\ , \tag{23}
$$

with the arguments,

$$
\begin{aligned}
D &= x_1x_3(1-r_2^2)\ , &(24)\\
D_1 &= (1-x_2)x_1r_2^2+(x_1+x_2-1)x_3(1-r_2^2)+\left[\frac{1}{4}-(1-x_2)^2\right]r_2^2\ , &(25)\\
D_2 &= x_1x_2r_2^2+(x_1-x_2)x_3(1-r_2^2)+\left(\frac{1}{4}-x_2^2\right)r_2^2\ . &(26)
\end{aligned}
$$

The hard scales t are chosen as

$$
t^{(j)}=\max(\sqrt{D}m_B,\sqrt{|D_j|}m_B,1/b_1)\ . \tag{27}
$$

Without the transverse momenta k_T, the terms containing $2r_Kr_2^2\Phi_K^\sigma\Psi^t$ in Eqs. (17) and (18) are logarithimically divergent due to the end-point singularities from $x_3\to 0$ [9]. Since the end-point singularities appear only at the power of $r_Kr_2^2$, instead of at leading power, the spectator contributions are expected to be evaluated more reliably than the factorizable contributions in PQCD.

For the $B\to J/\psi K^*$ modes, the parametrization of the kinematic variables in Eqs. (3) and (4) still apply. We further define the polarization vectors ϵ_3 for the K^* meson, which is orthogonal to the K^* meson momentum P_3. In this case the K^* meson mass should be kept in the intermediate stage of the calculation, and then dropped at the end according to the power counting rules in [27]. The decay rate is given by

$$
\Gamma=\frac{1}{32\pi}G_F^2|V_{cb}|^2|V_{cs}|^2m_B^3(1-r_2^2)^3\sum_\sigma\mathcal{A}^{(\sigma)\dagger}\mathcal{A}^{(\sigma)}\ . \tag{28}
$$

The amplitudes for different final helicity states are expressed as

$$
\begin{aligned}
\mathcal{A}^{(\sigma)} &= -\Big\{a_{\rm eff}(\mu) f_{J/\psi} F(m_{J/\psi}^2) + \mathcal{M}_L^{(J/\psi K^*)}, \\
&\quad \epsilon_{2T}^* \cdot \epsilon_{3T}^* \left[r_2 a_{\rm eff}(\mu) f_{J/\psi} A_1(m_{J/\psi}^2) + \mathcal{M}_N^{(J/\psi K^*)}\right], \\
&\quad i\,\epsilon^{\alpha\beta\gamma\rho}\epsilon_{2\alpha}^*\epsilon_{3\beta}^* \frac{P_{2\gamma}P_{3\rho}}{m_B^2}\left[r_2 a_{\rm eff}(\mu) f_{J/\psi} V(m_{J/\psi}^2) + \mathcal{M}_T^{(J/\psi K^*)}\right]\Big\}, \qquad (29)
\end{aligned}
$$

with the combination of the form factors,

$$
F(m_{J/\psi}^2) = \frac{1+r_{K^*}}{2r_{K^*}} A_1(m_{J/\psi}^2) - \frac{1-r_2^2}{2r_{K^*}(1+r_{K^*})} A_2(m_{J/\psi}^2)\ , \qquad (30)
$$

and the mass ratio $r_{K^*} = m_{K^*}/m_B$. The first term in Eq. (29) corresponds to the configuration with both the vector mesons being longitudinally polarized, and the second (third) term to the two configurations with both the vector mesons being transversely polarized in the parallel (perpendicular) directions.

The effective Wilson coefficient $a_{\rm eff}$ is the same as in Eq. (10). The $B \to K^*$ transition form factors A_1, A_2, and V are defined via the matrix elements,

$$
\begin{aligned}
\langle K^*(P_3,\epsilon_3)|\bar{b}\gamma^\mu s|B(P_1)\rangle &= \frac{2iV(q^2)}{m_B+m_{K^*}}\epsilon^{\mu\nu\rho\sigma}\epsilon_{3\nu}^* P_{3\rho}P_{1\sigma}, \qquad (31) \\
\langle K^*(P_3,\epsilon_3)|\bar{b}\gamma^\mu\gamma_5 s|B(P_1)\rangle &= 2m_{K^*}A_0(q^2)\frac{\epsilon_3^*\cdot q}{q^2}q^\mu + (m_B+m_{K^*})A_1(q^2)\left(\epsilon_3^{*\mu} - \frac{\epsilon_3^*\cdot q}{q^2}q^\mu\right) \\
&\quad - A_2(q^2)\frac{\epsilon_3^*\cdot q}{m_B+m_{K^*}}\left(P_1^\mu + P_3^\mu - \frac{m_B^2-m_{K^*}^2}{q^2}q^\mu\right)\ . \qquad (32)
\end{aligned}
$$

The spectator amplitudes $\mathcal{M}_{L,N,T}^{(J/\psi K^*)}$ from Figs. 1(e) and 1(f) are written as

$$
\mathcal{M}_{L,N,T}^{(J/\psi K^*)} = \mathcal{M}_{L4,N4,T4}^{(J/\psi K^*)} + \mathcal{M}_{L6,N6,T6}^{(J/\psi K^*)}\ , \qquad (33)
$$

where the explicit factorization formulas for the amplitudes $\mathcal{M}_{L4,N4,T4}^{(J/\psi K^*)}$ and for $\mathcal{M}_{L6,N6,T6}^{(J/\psi K^*)}$ are presented in the Appendix.

The helicity amplitudes are then defined as,

$$
\begin{aligned}
A_L &= G\left[a_{\rm eff}(\mu) f_{J/\psi} F(m_{J/\psi}^2) + \mathcal{M}_L^{(J/\psi K^*)}\right]\ , \\
A_\parallel &= -G\sqrt{2}\left[r_2 a_{\rm eff}(\mu) f_{J/\psi} A_1(m_{J/\psi}^2) + \mathcal{M}_N^{(J/\psi K^*)}\right]\ , \\
A_\perp &= -G r_2 r_{K^*}\sqrt{2[(v_2\cdot v_3)^2-1]}\left[r_2 a_{\rm eff}(\mu) f_{J/\psi} V(m_{J/\psi}^2) + \mathcal{M}_T^{(J/\psi K^*)}\right]\ , \qquad (34)
\end{aligned}
$$

for the longitudinal, parallel, and perpendicular polarizations, respectively, with the velocities $v_2 = P_2/m_{J/\psi}$ and $v_3 = P_3/m_{K^*}$. The normalization factor G has been chosen such that the following relation is satisfied:

$$
|A_L|^2 + |A_\parallel|^2 + |A_\perp|^2 = 1\ . \qquad (35)
$$

In our convention the relative phase $\delta_\parallel$ ($\delta_\perp$) between $A_\parallel$ ($A_\perp$) and A_L takes the value of π in the heavy-quark limit, ie., as $a_{\rm eff}$ is real and $\mathcal{M}_{L,N,T}^{(J/\psi K^*)}$ vanish. We define the polarization fractions $f_L = |A_L|^2$, $f_\parallel = |A_\parallel|^2$, and $f_\perp = |A_\perp|^2$. The $B \to J/\psi K^*$ polarizations, significantly affected by the nonfactorizable contributions, were not discussed in [33].

2.2 Numerical Analysis

To perform the numerical analysis, we need the information of the involved meson distribution amplitudes. The B meson wave function Φ_B and the $K^{(*)}$ meson distribution amplitudes have been studied intensively, whose expressions are collected in the Appendix. The charmonium distribution amplitudes attract less attention in the literature. In [34] they were assumed to be identical to the corresponding light-meson distribution amplitudes: those of the J/ψ (η_c) meson are the same as of the ρ (π) meson. Then the twist-3 charmonium distribution amplitudes do not vanish at the end points $x = 0$, 1, where the valence charm quarks become highly off-shell. Such a functional shape seems to contradict to the intuition. More realistic asymptotic models have been proposed recently in [19], which were inferred from the non-relativistic heavy quarkonium bound-state wave functions with the same quantum numbers. We shall adopt these J/ψ and η_c meson distribution amplitudes, and derive the χ_{c0} and χ_{c1} meson distribution amplitudes following the similar procedure in the next section.

The J/ψ meson asymptotic distribution amplitudes are given by [19]

$$\begin{aligned}
\Psi^L(x) &= \Psi^T(x) = 9.58\frac{f_{J/\psi}}{2\sqrt{2N_c}}x(1-x)\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}, \\
\Psi^t(x) &= 10.94\frac{f_{J/\psi}}{2\sqrt{2N_c}}(1-2x)^2\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}, \\
\Psi^V(x) &= 1.67\frac{f_{J/\psi}}{2\sqrt{2N_c}}\left[1+(2x-1)^2\right]\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7},
\end{aligned} \tag{36}$$

in which the twist-3 ones $\Psi^{t,V}$ vanish at the end points due to the additional factor $[x(1-x)]^{0.7}$. Compared to [20], we have distinguished the distribution amplitudes associated with the longitudinally and transversely polarized J/ψ mesons, which exhibit the different asymptotic behaviors. We do not distinguish the decay constant $f_{J/\psi}$ for the normalization of Ψ^L and $f^T_{J/\psi}$ for the normalization of Ψ^t. The distribution amplitudes associated with the structures I (the identity) and $\gamma^\mu\gamma_5$ (the pseudo-vector) diminish like $1 - 2m_c/m_{J/\psi}$, m_c being the c quark mass. The above models have been shown to yield the observed cross section of charmonium production in e^+e^- collisions [19]. If factorization theorem works, the same distribution amplitudes, due to their universality, should be able to explain the exclusive B meson decays into charmonia.

	$F(0)$	a	b
$F_1(q^2)$	0.35	1.58	0.68
$F_0(q^2)$	0.35	0.71	0.04
$V(q^2)$	0.31	1.79	1.18
$A_0(q^2)$	0.31	1.68	1.08
$A_1(q^2)$	0.26	0.93	0.19
$A_2(q^2)$	0.24	1.63	0.98

Table 1: Parameters for the $B \to K^{(*)}$ transition form factors.

$X(J^{PC})$	m_X[GeV][35]	f_X[MeV]
$\eta_c(0^{-+})$	2.980	420 ± 50 [36]
$J/\psi(1^{--})$	3.097	405 ± 14 [12]
$\chi_{c0}(0^{++})$	3.415	360 [37]
$\chi_{c1}(1^{++})$	3.511	335 [37]

Table 2: Decay constants and masses of various charmonium states.

For the $B \to K^{(*)}$ transition form factors, we employ the models derived from the light-front QCD [38], which have been parameterized as

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2} , \tag{37}$$

with the constants $F(0)$, a, and b being listed in Table 1. Using the inputs in Table 2 [39] for the various charmonium states, the CKM matrix elements $V_{cb} = 0.040$ and $V_{cs} = 0.996$, the quark mass $m_t = 174.3$ GeV for the Wilson coefficients, the lifetimes $\tau_{B^0} = 1.56 \times 10^{-12}$ sec and $\tau_{B^\pm} = 1.67 \times 10^{-12}$ sec, and the Fermi constant $G_F = 1.16639 \times 10^{-5}$ GeV^{-2}, we derive the branching ratios,

$$\begin{aligned}
B(B^+ \to J/\psi K^+) &= (9.20^{+6.03}_{-7.99}) \times 10^{-4} , \\
B(B^0 \to J/\psi K^0) &= (8.60^{+5.63}_{-7.47}) \times 10^{-4} , \\
B(B^+ \to J/\psi K^{*+}) &= (9.95^{+5.2}_{-7.16}) \times 10^{-4} , \\
B(B^0 \to J/\psi K^{*0}) &= (9.30^{+4.86}_{-6.69}) \times 10^{-4} .
\end{aligned} \tag{38}$$

We have also derived the the polarization fractions and the relative phases among the helicity amplitudes for the $B \to J/\psi K^*$ decays,

$$\begin{aligned}
&f_L = 0.73^{+0.15}_{-0.05}, \quad f_\parallel = 0.17^{+0.04}_{-0.10}, \quad f_\perp = 0.10^{+0.01}_{-0.05} , \\
&\delta_\parallel = 2.57^{+0.12}_{-0.93} , \quad \delta_\perp = 2.53^{+0.13}_{-0.90} .
\end{aligned} \tag{39}$$

Both the $B^+ \to J/\psi K^{*0+}$ and $B^0 \to J/\psi K^{*0}$ modes have the same polarization fractions, since they differ only in the lifetimes in our formalism. The theoretical uncertainties come from the variation of the renormalization scale μ. We have checked the sensitivity of our results to the choice of the b quark mass: $m_b = 4.8$ GeV increases the branching ratio $B(B^0 \to J/\psi K^0)$ only by 10%.

The recent Babar measurement gave [5]

$$\begin{aligned}
B(B^+ \to J/\psi K^{*+}) &= (14.54 \pm 0.47 \pm 0.97) \times 10^{-4} , \\
B(B^0 \to J/\psi K^{*0}) &= (13.09 \pm 0.26 \pm 0.77) \times 10^{-4}
\end{aligned} \tag{40}$$

and the Belle measurement gave [40]

$$\begin{aligned}
&f_L = 0.585 \pm 0.012 \pm 0.009, \quad f_\parallel = 0.233 \pm 0.013 \pm 0.008, \quad f_\perp = 0.181 \pm 0.012 \pm 0.008 , \\
&\delta_\parallel = 2.888 \pm 0.090 \pm 0.008 , \quad \delta_\perp = 2.903 \pm 0.064 \pm 0.010 .
\end{aligned} \tag{41}$$

It is found that the consistency between the theoretical and experimental values for the $B \to J/\psi K^{(*)}$ branching ratios is reasonable. The predicted longitudinal polarization fraction f_L for the $B \to J/\psi K^*$ is a bit larger, and the predicted relative phases are a bit smaller than the data. However, we point out that the polarization fractions and the relative phases could be modified by adding the higher Gegenbauer terms to the J/ψ meson distribution amplitudes, and that the consistency will be improved. This fine tuning will not be performed here. Roughly speaking, it is very promising to understand the $B \to J/\psi K^{(*)}$ data in the PQCD framework.

Another remark is as follows. Most of the J/ψ meson distribution amplitudes, except Ψ^t, exhibit maxima at $x = 1/2$, such that the valence charm quark, carrying the invariant mass $x^2P_2^2 \approx m_c^2$, is almost on-shell. However, Ψ^t has two humps with a dip at $x = 1/2$. We argue that the models in Eq. (36) are the asymptotic ones. Including the higher Gegenbauer terms, such as those for a light vector meson,

$$\begin{aligned}\Psi^t(x) &= 10.94\frac{f_{J/\psi}}{2\sqrt{2N_c}}\Big\{(1-2x)^2 + a_1^{J/\psi}(2x-1)^2[5(2x-1)^2-3] \\ &\quad + a_2^{J/\psi}[3-30(2x-1)^2+35(2x-1)^4]\Big\}\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.70}, \end{aligned} \tag{42}$$

Ψ^t possess three humps with the major one located at $x = 1/2$. Adopting this parametrization, the Gegenbauer coefficients $a_1^{J/\psi}$ and $a_2^{J/\psi}$ should be determined from the data of charmonium production in e^+e^- annihilation, before it is applied to the exclusive B meson decays into charmonia.

3 $B \to (\chi_{c0}, \chi_{c1}, \eta_c)K^{(*)}$ DECAYS

Before calculating the spectator contributions to other similar modes, we clarify a confusing statement in the literature. It has been claimed [10] that the end-point singularities from $x_3 \to 0$ cancel between the spectator diagrams Figs. 1(e) and 1(f) in the QCDF formalism for the $B \to J/\psi K$ decays, but do not for the $B \to \chi_{c1} K$ decays. Our opinion is that the end-point singularities should appear in both modes at the power of r_2^2, if no approximation is made. The singularities cancel in [9] because of the relation,

$$\frac{f_{J/\psi}^T m_c}{f_{J/\psi} m_{J/\psi}} = 2x_2^2 , \tag{43}$$

where the decay constant $f_{J/\psi}^T$ is associated with the normalization of the twist-3 distribution amplitude Ψ^t. This relation arises from the exact on-shell condition of the valence charm quarks. The singularities were present in [10], since the authors, considering only the leading-twist χ_{c1} meson distribution amplitude, got no chance to apply Eq. (43). We agree on the comment made in [9]: it is not clear why the singularities cancel in [7] for the $B \to J/\psi K$ decays, viewing that they treated the ratio on the left-hand side of Eq. (43) as a constant. The exact on-shell condition is indeed not necessary, because a deviation from mass shell, as long as being power-suppressed, is allowed. Furthermore, the momentum fraction x_2, running between 0 and 1, is hardly related to the left-hand side. Once Eq. (43) is not postulated, the end-point singularities exist in the terms proportional to r_2^2 for the $B \to J/\psi K$ decays [20], and also for

the $B \to \chi_{c1} K$ decays. Note that these singularities disappear in the PQCD approach based on k_T factorization theorem.

The PQCD analysis of the $B \to \chi_{c1} K^{(*)}$ decays is similar to that of $B \to J/\psi K^{(*)}$. To calculate the nonfactorizable spectator amplitudes, we consider the χ_{c1} meson distribution amplitudes defined via the nonlocal matrix elements associated with the longitudinal and transverse polarizations,

$$\langle \chi_{c1}(P,\epsilon_L)|\bar{c}(z)_j c(0)_l|0\rangle = \frac{1}{\sqrt{2N_c}}\int_0^1 dx e^{ixP\cdot z}\Big\{m_{\chi_{c1}}[\gamma_5 \not\epsilon_L]_{lj}\chi_1^L(x) + [\gamma_5 \not\epsilon_L \not P]_{lj}\chi_1^t(x)\Big\}, \quad (44)$$

$$\langle \chi_{c1}(P,\epsilon_T)|\bar{c}(z)_j c(0)_l|0\rangle = \frac{1}{\sqrt{2N_c}}\int_0^1 dx e^{ixP\cdot z}\Big\{m_{\chi_{c1}}[\gamma_5 \not\epsilon_T]_{lj}\chi_1^V(x) + [\gamma_5 \not\epsilon_T \not P]_{lj}\chi_1^T(x)\Big\} \quad (45)$$

respectively, with the χ_{c1} meson mass $m_{\chi_{c1}}$. The asymptotic models for the twist-2 distribution amplitudes $\chi_1^L(x)$ and $\chi_1^T(x)$, and for the twist-3 distribution amplitudes $\chi_1^t(x)$ and $\chi_1^V(x)$ will be derived following the prescription in [19]: they can be extracted from the $n = 2$, $l = 1$ Schrodinger states for a Coulomb potential, whose radial dependence is given by

$$\Psi_{\rm Sch}(r) \propto r \exp(-q_B r) , \quad (46)$$

q_B being the Bohr momentum. Fourier transformation of the above solution leads to

$$\Psi_{\rm Sch}(k) \propto \frac{k^2 - 3q_B^2}{(k^2 + q_B^2)^3} , \quad (47)$$

with $k^2 = |\mathbf{k}|^2$. Employing the substitution [19],

$$\mathbf{k}_T \to \mathbf{k}_T , \quad k_z \to (x - \bar{x})\frac{m_D}{2} , \quad m_D^2 = \frac{m_c^2 + k_T^2}{x\bar{x}} , \quad (48)$$

x ($\bar{x} \equiv 1 - x$) being the c ($\bar{c}$) quark momentum fraction, we obtain the heavy quarkonium distribution amplitude,

$$\Phi(x) \sim \int d^2 k_T \Psi_{\rm Sch}(x, k_T) \propto x\bar{x}\left\{\frac{x\bar{x}[1 - 4x\bar{x}(1 + v^2)]}{[1 - 4x\bar{x}(1 - v^2)]^2}\right\} , \quad (49)$$

with $v^2 = q_B^2/m_c^2$.

To impose a fractional power $1 - v^2$ to the factor in the curved brackets [19], we neglect the v^2 term in the numerator. Otherwise, the numerator may become negative for

$$1 - \sqrt{1 - \frac{1}{1 + v^2}} < 2x < 1 + \sqrt{1 - \frac{1}{1 + v^2}} , \quad (50)$$

and the analyticity of the distribution amplitude will be lost. We then propose the χ_{c1} meson distribution amplitudes inferred from Eq. (49),

$$\chi_1(x) \propto \Phi^{\rm asy}(x)\left\{\frac{x\bar{x}(1 - 4x\bar{x})}{[1 - 4x\bar{x}(1 - v^2)]^2}\right\}^{1-v^2} , \quad (51)$$

with $\Phi^{\rm asy}(x)$ being set to the asymptotic models of the corresponding twists for light vector mesons. It is now clear that after imposing the fractional power $1 - v^2$, Eq. (51) approaches

the light meson distribution amplitudes as $v^2 \to 1$, and the heavy quarkonium distribution amplitudes the same as Eq. (49) as $v^2 \to 0$ [19]. Fixing the normalization, we derive

$$\begin{aligned}
\chi_1^L(x) &= \chi_1^T(x) = 27.46\frac{f_{\chi_{c1}}}{2\sqrt{2N_c}}x(1-x)\left\{\frac{x(1-x)[1-4x(1-x)]}{[1-2.8x(1-x)]^2}\right\}^{0.7}, \\
\chi_1^t(x) &= 15.17\frac{f_{\chi_{c1}}}{2\sqrt{2N_c}}(1-2x)^2\left\{\frac{x(1-x)[1-4x(1-x)]}{[1-2.8x(1-x)]^2}\right\}^{0.7}, \\
\chi_1^V(x) &= 3.60\frac{f_{\chi_{c1}}}{2\sqrt{2N_c}}\left[1+(2x-1)^2\right]\left\{\frac{x(1-x)[1-4x(1-x)]}{[1-2.8x(1-x)]^2}\right\}^{0.7}, \qquad (52)
\end{aligned}$$

where the same decay constant $f_{\chi_{c1}}$ has been assumed for the longitudinally and transversely polarized χ_{c1} meson. Note that the dip at $x = 1/2$ is a consequence of the P-wave Schrodinger wave functions.

The $B \to \chi_{c1}K^{(*)}$ decay amplitudes are written as

$$\mathcal{A} = a_{\rm eff}(\mu)f_{\chi_{c1}}F_1(m_{\chi_{c1}}^2) + \mathcal{M}^{(\chi_{c1}K)}, \qquad (53)$$

$$\begin{aligned}
\mathcal{A}^{(\sigma)} = -\Big\{ & a_{\rm eff}(\mu)f_{\chi_{c1}}F(m_{\chi_{c1}}^2) + \mathcal{M}_L^{(\chi_{c1}K^*)}, \\
& \epsilon_{2T}^*\cdot\epsilon_{3T}^*\left[a_{\rm eff}(\mu)r_2f_{\chi_{c1}}A_1(m_{\chi_{c1}}^2) + \mathcal{M}_N^{(\chi_{c1}K^*)}\right], \\
& i\,\epsilon^{\alpha\beta\gamma\rho}\epsilon_{2\alpha}^*\epsilon_{3\beta}^*\frac{P_{2\gamma}P_{3\rho}}{m_B^2}\left[r_2f_{\chi_{c1}}V(m_{\chi_{c1}}^2) + \mathcal{M}_T^{(\chi_{c1}K^*)}\right]\Big\}, \qquad (54)
\end{aligned}$$

with the mass ratio $r_2 = m_{\chi_{c1}}/m_B$, and the effective Wilson coefficient,

$$a_{\rm eff}(\mu) = a_2(\mu) + a_3(\mu) - a_5(\mu). \qquad (55)$$

Similarly, we shall vary the renormalization scale μ in the range between $0.5m_b$ and $1.5m_b$, which covers the effect of the vertex corrections. As in the case of $B \to J/\psi K$, the infrared divergences in the vertex corrections cancel in the $B \to \chi_{c1}K$ decays: the hard kernel of the vertex corrections is odd for the structure $\gamma_5\ \not{\epsilon}$ under the exchange of x and $1-x$, while the corresponding distribution amplitude is even. The hard kernel vanishes for the structure $\gamma_5\ \not{\epsilon}\ \not{P}$. Express the spectator amplitudes $\mathcal{M}^{(\chi_{c1}K)}$ and $\mathcal{M}_{L,N,T}^{(\chi_{c1}K^*)}$ as

$$\begin{aligned}
\mathcal{M}^{(\chi_{c1}K)} &= \mathcal{M}_4^{(\chi_{c1}K)} + \mathcal{M}_6^{(\chi_{c1}K)}, \\
\mathcal{M}_{L,N,T}^{(\chi_{c1}K^*)} &= \mathcal{M}_{L4,N4,T4}^{(\chi_{c1}K^*)} + \mathcal{M}_{L6,N6,T6}^{(\chi_{c1}K^*)}, \qquad (56)
\end{aligned}$$

where the factorization formulas of the amplitudes $\mathcal{M}_4^{(\chi_{c1}K)}$, $\mathcal{M}_6^{(\chi_{c1}K)}$, $\mathcal{M}_{L4,N4,T4}^{(\chi_{c1}K^*)}$ and $\mathcal{M}_{L6,N6,T6}^{(\chi_{c1}K^*)}$ are shown in the Appendix. Their expressions are similar to those of $B \to J/\psi K^{(*)}$ with some terms flipping signs.

Using the inputs listed in Table 2 [39], we get

$$\begin{aligned}
& B(B^+ \to \chi_{c1}K^+) = (3.15^{+3.17}_{-2.61})\times 10^{-4}, \\
& B(B^0 \to \chi_{c1}K^0) = (2.94^{+2.97}_{-2.43})\times 10^{-4}, \\
& B(B^+ \to \chi_{c1}K^{*0+}) = (2.99^{+0.40}_{-0.50})\times 10^{-3}, \\
& B(B^0 \to \chi_{c1}K^{*0}) = (2.79^{+0.37}_{-0.46})\times 10^{-3}, \\
& f_L = 0.38^{+0.03}_{-0.08},\quad f_\parallel = 0.07^{+0.01}_{-0.01},\quad f_\perp = 0.55^{+0.06}_{-0.05}, \\
& \delta_\parallel = 2.61^{+0.16}_{-0.43},\ \ \delta_\perp = 2.58^{+0.07}_{-0.03}. \qquad (57)
\end{aligned}$$

The errors of the above predictions arise from the variation of the renormalization scale μ for the factorizable contribution. For the $B \to \chi_{c1} K^*$ branching ratios, the imaginary part of the spectator amplitude $\mathcal{M}_T^{(\chi_{c1}K^*)}$ dominates, such that the influence of the μ dependence becomes mild. This is also the reason we obtain a large $f_\perp$. The $B \to \chi_{c1} K$ branching ratios in Eq. (57) are consistent with the recent Babar measurement [5],

$$
\begin{aligned}
B(B^+ \to \chi_{c1} K^+) &= (5.79 \pm 0.26 \pm 0.65) \times 10^{-4} , \\
B(B^0 \to \chi_{c1} K^0) &= (4.53 \pm 0.41 \pm 0.51) \times 10^{-4} .
\end{aligned} \tag{58}
$$

Note that these branching ratios, with a parametrization for the logarithmical end-point singularities, were estimated to be only about 10^{-4} in QCDF [10]. Our predictions for the $B \to \chi_{c1} K^*$ decays, including the branching ratios, the polarization fractions (especially the large $f_\perp$), and the relative phases, can be compared with data in the future.

We then discuss the $B \to \chi_{c0} K$ decays, for which the nonfactorizable contributions dominate due to the absence of the factorizable ones. The nonlocal matrix element associated with the χ_{c0} meson is decomposed into,

$$
\langle \chi_{c0}(P)|\bar{c}(z)_j c(0)_l|0\rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP\cdot z} \left\{ [\not{P}]_{lj} \chi_0^v(x) + m_{\chi_{c0}} [I]_{lj} \chi_0^s(x) \right\} , \tag{59}
$$

which defines the twist-2 and twist-3 distribution amplitudes, $\chi_0^v(x)$ and $\chi_0^s(x)$, respectively, $m_{\chi_{c0}}$ being the χ_{c0} meson mass. To satisfy the identity $\langle \chi_{c0}(P)|\bar{c}\gamma_\mu c|0\rangle = 0$, $\chi_0^v(x)$ must be anti-symmetric under the exchange of x and $1-x$. Following the similar ansatz of constructing the χ_{c1} meson distribution amplitudes, we propose the following asymptotic models,

$$
\begin{aligned}
\chi_0^v(x) &= 27.46 \frac{f_{\chi_{c0}}}{12\sqrt{2N_c}} (1-2x) \left\{ \frac{x(1-x)[1-4x(1-x)]}{[1-2.8x(1-x)]^2} \right\}^{0.7} , \\
\chi_0^s(x) &= 4.73 \frac{f_{\chi_{c0}}}{2\sqrt{2N_c}} \left\{ \frac{x(1-x)[1-4x(1-x)]}{[1-2.8x(1-x)]^2} \right\}^{0.7} ,
\end{aligned} \tag{60}
$$

where the decay constant $f_{\chi_{c0}}$ for the normalization of χ_0^v and χ_0^s has been assumed to be equal.

In the QCDF analysis of the $B \to (J/\psi, \chi_{c1}) K^{(*)}$ decays, the vertex corrections from Figs. 1(a)-1(d) are infrared safe. However, for the $B \to \chi_{c0} K$ decays, the hard kernels of the vertex corrections are even for the structure I, and odd for $\not{P}$ under the exchange of x and $1-x$. The corresponding χ_{c0} meson distribution amplitudes are also even for the structure I, and odd for $\not{P}$. Therefore, the infrared divergences do not cancel [10]. To regularize these divergences, a binding energy $2m_c - m_{\chi_{c0}}$ and a gluon mass m_g have been introduced in [41] and [42], respectively. The vertex corrections to the $B \to \chi_{c0} K$ decays can also be handled in the PQCD approach. For the reasons stated in the Introduction, we do not attempt such a calculation here. Instead, we shall demonstrate that the spectator contributions are sufficient to account for the observed $B \to \chi_{c0} K$ branching ratios. The decay amplitudes are written as

$$
\mathcal{A} = \mathcal{M}^{(\chi_{c0}K^{(*)})} , \quad \mathcal{M}^{(\chi_{c0}K^{(*)})} = \mathcal{M}_4^{(\chi_{c0}K^{(*)})} + \mathcal{M}_6^{(\chi_{c0}K^{(*)})} , \tag{61}
$$

with the explicit factorization formulas of the amplitudes $\mathcal{M}_{4,6}^{(\chi_{c0}K^{(*)})}$ being referred to the Appendix. There are also logarithmic end-point singularities in these amplitudes, if using QCDF, indicating that the PQCD approach based on k_T factorization theorem is more appropriate.

Adopting the inputs in Table 2, we obtain the branching ratios,

$$
\begin{aligned}
B(B^+ \to \chi_{c0} K^+) &= 5.61 \times 10^{-4} \,, \\
B(B^0 \to \chi_{c0} K^0) &= 5.24 \times 10^{-4} \,, \\
B(B^+ \to \chi_{c0} K^{*+}) &= 8.69 \times 10^{-4} \,, \\
B(B^0 \to \chi_{c0} K^{*0}) &= 8.12 \times 10^{-4} \,.
\end{aligned} \tag{62}
$$

There is no theoretical uncertainty from the variation of μ, since the factorizable contributions vanish in this case. Babar gave the upper bounds (the values) [43],

$$
\begin{aligned}
B(B^+ \to \chi_{c0} K^+) &< 8.9 \;\; (= 4.4 \pm 3.3 \pm 0.7) \times 10^{-4} \,, \\
B(B^0 \to \chi_{c0} K^0) &< 12.4 \;\; (= 5.3 \pm 5.0 \pm 0.8) \times 10^{-4} \,,
\end{aligned} \tag{63}
$$

and Belle gave [44]

$$
B(B^+ \to \chi_{c0} K^+) = (1.96 \pm 0.35 \pm 0.33^{+1.97}_{-0.26}) \times 10^{-4} \,, \tag{64}
$$

where the third error comes from a model uncertainty. Hence, we conclude that the above data are understandable. The $B^+ \to \chi_{c0} K^+$ branching ratio was found to be as small as 4.2×10^{-5} in [41] (with a large theoretical uncertainty from parameterizing the end-point singularities in QCDF) and $(2-3) \times 10^{-4}$ for the gluon mass $m_g = 0.5 - 0.2$ GeV in [42]. Despite of several discrepancies in their factorization formulas for the vertex corrections and for the spectator amplitudes, the difference in their numerical results is not really essential due to the tunable parameters.

The analysis of the $B \to \eta_c K^{(*)}$ decays is similar to that of $B \to \chi_{c0} K^{(*)}$. The nonlocal matrix element associated with the χ_{c0} meson is decomposed into

$$
\langle \eta_c(P) | \bar{c}(z)_j c(0)_l | 0 \rangle \;=\; \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP\cdot z} \Big\{ [\gamma_5 \; \not{P}]_{lj} \eta^v(x) + m_{\eta_c} [\gamma_5]_{lj} \eta^s(x) \Big\} \,, \tag{65}
$$

which defines the twist-2 and twist-3 η_c meson distribution amplitudes, $\eta^v(x)$ and $\eta^s(x)$, respectively, m_{η_c} being the η_c meson mass. The asymptotic models for the η_c meson distribution amplitudes have been determined in [19]:

$$
\begin{aligned}
\eta^v(x) &= 9.58 \frac{f_{\eta_c}}{2\sqrt{2N_c}} x(1-x) \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7} \,, \\
\eta^s(x) &= 1.97 \frac{f_{\eta_c}}{2\sqrt{2N_c}} \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7} \,,
\end{aligned} \tag{66}
$$

where we have assumed the same the decay constant f_{η_c} for the normalization of η^v and η^s.

The decay amplitudes are written as

$$
\mathcal{A} = a_{\rm eff}(\mu) f_{\eta_c} F_0(m_\eta^2) + \mathcal{M}^{(\eta_c K)} \,, \quad \mathcal{A} = a_{\rm eff}(\mu) f_{\eta_c} A_0(m_\eta^2) + \mathcal{M}^{(\eta_c K^*)} \,, \tag{67}
$$

for which the factorization formulas of the spectator amplitudes,

$$
\mathcal{M}^{(\eta_c K^{(*)})} = \mathcal{M}_4^{(\eta_c K^{(*)})} + \mathcal{M}_6^{(\eta_c K^{(*)})} \,, \tag{68}
$$

can be found in the Appendix. The $B \to K$ transition form factor F_0 and the $B \to K^*$ transition form factor A_0 have been defined in Eqs. (9) and (32), respectively. The effective Wilson coefficient $a_{\rm eff}(\mu)$ for the $B \to \chi_{c0} K^{(*)}$ decays is also given by Eq. (55). Similarly, we shall vary the renormalization scale μ in the range between $0.5m_b$ and $1.5m_b$, which covers the effect of the vertex corrections. The hard kernels of the vertex corrections are even for the structure γ_5, and odd for $\gamma_5 \not{P}$ under the exchange of x and $1-x$. The corresponding η_c meson distribution amplitudes are both even for γ_5 and $\gamma_5 \not{P}$. Therefore, the infrared divergences in the vertex corrections to the $B \to \eta_c K$ decay, cancelling only for $\gamma_5 \not{P}$, still exist.

Using the inputs in Table 2 [39] and the form factor parametrization in Eq. (37), we obtain the branching ratios,

$$\begin{aligned} B(B^+ \to \eta_{c0} K^+) &= (2.34^{+2.43}_{-2.11}) \times 10^{-4} \;, \\ B(B^0 \to \eta_{c0} K^0) &= (2.19^{+2.13}_{-2.12}) \times 10^{-4} \;, \\ B(B^+ \to \eta_c K^{*+}) &= (2.82^{+2.91}_{-2.76}) \times 10^{-4} \;, \\ B(B^0 \to \eta_c K^{*0}) &= (2.64^{+2.71}_{-2.58}) \times 10^{-4} \;. \end{aligned} \tag{69}$$

Since the η_c meson distribution amplitudes have no two humps, the $B \to \eta_c K$ branching ratios are expected to be smaller than the $B \to J/\psi K$ ones. However, the Belle measurement [45],

$$\begin{aligned} B(B^+ \to \eta_c K^+) &= (1.25 \pm 0.14^{+0.10}_{-0.12} \pm 0.38) \times 10^{-3} \;, \\ B(B^0 \to \eta_c K^0) &= (1.23 \pm 0.23^{+0.12}_{-0.16} \pm 0.38) \times 10^{-3} \;, \end{aligned} \tag{70}$$

and the Babar measurement [46],

$$\begin{aligned} B(B^+ \to \eta_c K^+) &= (1.34 \pm 0.09 \pm 0.13 \pm 0.41) \times 10^{-3} \;, \\ B(B^0 \to \eta_c K^0) &= (1.18 \pm 0.16 \pm 0.13 \pm 0.37) \times 10^{-3} \;, \end{aligned} \tag{71}$$

are significantly larger than our predictions. The QCDF approach gave the $B \to \eta_c K$ branching ratios about $(1.4 - 1.9) \times 10^{-4}$ for a rough estimate, which are also too small [11].

Because the η_c meson distribution amplitudes in Eq. (66) have been shown to produce the observed cross section for charmonium production in e^+e^- annihilation [19], they are supposed to explain the branching ratios of the exclusive B meson decays into charmonia according to the universality, even though these two processes involve dramatically different dynamics. Therefore, we conclude that the $B \to \eta_c K$ decays are the only puzzle, and demand more studies.

4 CONCLUSION

The exclusive B meson decays into charmonia, $B \to XK^{(*)}$, for $X = J/\psi, \chi_{c0}, \chi_{c1}$, and η_c have been analyzed in the QCDF approach in the literature. Among these modes, those with $X = \chi_{c0}$ were claimed to be explainable, and those with $X = J/\psi, \chi_{c1}$, and η_c were not. However, this conclusion is not solid due to the serious end-point singularities in many QCDF decay amplitudes for the vertex corrections and for the spectator contributions, and due to the ad-hoc models of the charmonium distribution amplitudes. In this paper we have investigated these decays in an improved framework: the factorizable contributions are still treated in FA, since the involved $B \to K^{(*)}$ transition form factors, evaluated at the charmonium mass, may not be

calculated perturbatively; the effect of the vertex corrections was taken into account by varying the renormalization scale μ, because their estimation in the PQCD approach needs additional nonperturbative information, i.e., the distribution of the charm quark in its transverse degrees of freedom inside a charmonium; the spectator contributions were computed in PQCD, in which the end-point singularities are smeared by the Sudakov factor associated with the $K^{(*)}$ meson. The J/ψ and η_c meson distribution amplitudes have been inferred from the $n = 1$, $l = 0$ Schrodinger state for a Coulomb potential [19], which produce the measured cross section of charmonium production in e^+e^- collisions. To analyze the $B \to (\chi_{c0}, \chi_{c1})K^{(*)}$ decays, we have obtained the χ_{c0}, χ_{c1} meson distribution amplitudes from the $n = 2$, $l = 1$ Schrodinger states following the similar procedure. That is, our models for the charmonium distribution amplitudes have been constrained theoretically and experimentally.

Our investigation has indicated that only the $B \to \eta_c K$ decays exhibit a puzzle, whose branching ratios are significantly smaller than the observed values. The data of the $B \to (J/\psi, \chi_{c0}, \chi_{c1})K$, $J/\psi K^*$ decays, including the branching ratios, the polarization fractions, and the relative phases among the various helicity amplitudes, are all understandable within theoretical uncertainty. If QCD factorization theorem works, the η_c meson distribution amplitudes, explaining the charmonium production data, are supposed to explain the exclusive B meson decays into charmonia. Hence, the $B \to \eta_c K$ modes require a more thorough study. We have predicted the branching ratios, the polarization fractions, and the relative phases associated with the $B \to (\chi_{c1}, \chi_{c0}, \eta_c)K^*$ decays, which can be compared with future measurements.

We thank K.T. Chao and H.Y. Cheng for useful discussions. This work was supported by the National Science Council of R.O.C. under Grant No. NSC-93-2112-M-001-014.

A FACTORIZATION FORMULAS

For the B meson wave function, we adopt the model [16],

$$\Phi_B(x,b) = N_B x^2(1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{x m_B}{\omega_B}\right)^2 - \frac{\omega_B^2 b^2}{2}\right] , \tag{72}$$

with the shape parameter $\omega_B = 0.4$ GeV. The normalization constant $N_B = 91.784$ GeV is related to the decay constant $f_B = 190$ MeV (in the convention $f_\pi = 130$ MeV). The K and K^* meson distribution amplitudes have been derived from QCD sum rules [23, 24],

$$\begin{aligned}
\Phi_K(x) &= \frac{3f_K}{\sqrt{2N_c}} x(1-x)\left\{1+0.51(1-2x)+0.3[5(1-2x)^2-1]\right\} , & (73)\\
\Phi_K^p(x) &= \frac{f_K}{2\sqrt{2N_c}}\left[1+0.24C_2^{1/2}(1-2x)-0.11C_4^{1/2}(1-2x)\right] , & (74)\\
\Phi_K^\sigma(x) &= \frac{f_K}{2\sqrt{2N_c}}(1-2x)\left[1+0.35(10x^2-10x+1)\right] , & (75)\\
\Phi_{K^*}(x) &= \frac{3f_{K^*}}{\sqrt{2N_c}} x(1-x)\left[1+0.57(1-2x)+0.07C_2^{3/2}(1-2x)\right] , & (76)\\
\Phi_{K^*}^t(x) &= \frac{f_{K^*}^T}{2\sqrt{2N_c}}\left\{0.3(1-2x)\left[3(1-2x)^2+10(1-2x)-1\right]+1.68C_4^{1/2}(1-2x)\right.
\end{aligned}$$

$$+0.06(1-2x)^2\left[5(1-2x)^2-3\right]+0.36\left[1-2(1-2x)(1+\ln(1-x))\right]\Big\}\ , \tag{77}$$

$$\Phi^s_{K^*}(x) = \frac{f^T_{K^*}}{2\sqrt{2N_c}}\Big\{3(1-2x)\left[1+0.2(1-2x)+0.6(10x^2-10x+1)\right] -0.12x(1-x)+0.36[1-6x-2\ln(1-x)]\Big\}\ , \tag{78}$$

$$\Phi^T_{K^*}(x) = \frac{3f^T_{K^*}}{\sqrt{2N_c}}x(1-x)\Big[1+0.6(1-2x)+0.04C_2^{3/2}(1-2x)\Big]\ , \tag{79}$$

$$\Phi^v_{K^*}(x) = \frac{f_{K^*}}{2\sqrt{2N_c}}\Big\{\frac{3}{4}\Big[1+(1-2x)^2+0.44(1-2x)^3\Big]+0.4C_2^{1/2}(1-2x) +0.88C_4^{1/2}(1-2x)+0.48[2x+\ln(1-x)]\Big\}\ , \tag{80}$$

$$\Phi^a_{K^*}(x) = \frac{f_{K^*}}{4\sqrt{2N_c}}\Big\{3(1-2x)\Big[1+0.19(1-2x)+0.81(10x^2-10x+1)\Big] -1.14x(1-x)+0.48[1-6x-2\ln(1-x)]\Big\}\ , \tag{81}$$

with the Gegenbauer polynomials

$$C_2^{1/2}(\xi)=\frac{1}{2}(3\xi^2-1)\ ,\quad C_4^{1/2}(\xi)=\frac{1}{8}(35\xi^4-30\xi^2+3)\ ,\quad C_2^{3/2}(\xi)=\frac{3}{2}(5\xi^2-1)\ , \tag{82}$$

and the decay constants $f_K = 160$ MeV, $f_{K^*} = 200$ MeV, and $f^T_{K^*} = 160$ MeV. The coefficients of the Gegenbauer polynomials correspond to the masses $m_K = 0.49$ GeV and $m_0^K = 1.7$ GeV. We adopt the K^* meson mass $m_{K^*} = 0.89$ GeV.

A.1 $B \to (J/\Psi, \chi_{c1})K^{(*)}$

We present the spectator amplitudes for the $B \to (J/\psi, \chi_{c1})K^{(*)}$ decays below, where the symbol X represents the charmonium J/ψ or χ_{c1}:

$$\begin{aligned}
\mathcal{M}_4^{XK} = {}& 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1) \\
&\times\Big\{\Big[(1-2r_2^2)(1-x_2)\Phi_K(x_3)X^L(x_2)+\zeta_Y\frac{1}{2}r_2^2\Phi_K(x_3)X^t(x_2) \\
&-r_K(1-r_2^2)x_3\Phi_K^p(x_3)X^L(x_2)+r_K\left(2r_2^2(1-x_2)+(1-r_2^2)x_3\right)\Phi_K^\sigma(x_3)X^L(x_2)\Big] \\
&\times E_4(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1) \\
&-\Big[(x_2+(1-2r_2^2)x_3)\Phi_K(x_3)X^L(x_2)+r_2^2(2r_K\Phi_K^\sigma(x_3)-\frac{1}{2}\Phi_K(x_3))X^t(x_2) \\
&-r_K(1-r_2^2)x_3\Phi_K^p(x_3)X^L(x_2)-r_K\left(2r_2^2x_2+(1-r_2^2)x_3\right)\Phi_K^\sigma(x_3)X^L(x_2)\Big] \\
&\times E_4(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\ ,
\end{aligned} \tag{83}$$

$$\mathcal{M}_6^{XK} = \zeta_X 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1)$$

$$
\begin{aligned}
&\times\Big\{\Big[(1-x_2+(1-2r_2^2)x_3)\Phi_K(x_3)X^L(x_2)+\zeta_X r_2^2(2r_K\Phi_K^\sigma(x_3)-\frac{1}{2}\Phi_K(x_3))X^t(x_2) \\
&-r_K(1-r_2^2)x_3\Phi_K^p(x_3)X^L(x_2)-r_K\left(2r_2^2(1-x_2)+(1-r_2^2)x_3\right)\Phi_K^\sigma(x_3)X^L(x_2)\Big] \\
&\times E_6(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1) \\
&-\Big[(1-2r_2^2)x_2\Phi_K(x_3)X^L(x_2)+\frac{1}{2}r_2^2\Phi_K(x_3)X^t(x_2) \\
&-r_K(1-r_2^2)x_3\Phi_K^p(x_3)X^L(x_2)+r_K\left(2r_2^2x_2+(1-r_2^2)x_3\right)\Phi_K^\sigma(x_3)X^L(x_2)\Big] \\
&\times E_6(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\;, \qquad (84)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{L4}^{XK^*} &= 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1) \\
&\times\Big\{\Big[(1-2r_2^2)(1-x_2)\Phi_{K^*}(x_3)X^L(x_2)+\zeta_X\frac{1}{2}r_2^2\Phi_{K^*}(x_3)X^t(x_2) \\
&-r_{K^*}(1-r_2^2)x_3\Phi_{K^*}^s(x_3)X^L(x_2)+r_{K^*}\left(2r_2^2(1-x_2)+(1-2r_2^2)x_3\right)\Phi_{K^*}^t(x_3)X^L(x_2)\Big] \\
&\times E_4(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1) \\
&-\Big[(x_2+(1-2r_2^2)x_3)\Phi_{K^*}(x_3)X^L(x_2)+r_2^2(2r_{K^*}\Phi_{K^*}^t(x_3)-\frac{1}{2}\Phi_{K^*}(x_3))X^t(x_2) \\
&-r_{K^*}(1-r_2^2)x_3\Phi_{K^*}^s(x_3)X^L(x_2)-r_{K^*}\left(2r_2^2x_2+(1-2r_2^2)x_3\right)\Phi_{K^*}^t(x_3)X^L(x_2)\Big] \\
&\times E_4(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\;, \qquad (85)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{L6}^{XK^*} &= \zeta_X 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1) \\
&\times\Big\{\Big[(1-x_2+(1-2r_2^2)x_3)\Phi_{K^*}(x_3)X^L(x_2)+\zeta_X r_2^2(2r_{K^*}\Phi_{K^*}^t(x_3)-\frac{1}{2}\Phi_{K^*}(x_3))X^t(x_2) \\
&-r_{K^*}(1-r_2^2)x_3\Phi_{K^*}^s(x_3)X^L(x_2)-r_{K^*}\left(2r_2^2(1-x_2)+(1-2r_2^2)x_3\right)\Phi_{K^*}^t(x_3)X^L(x_2)\Big] \\
&\times E_6(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1) \\
&-\Big[(1-2r_2^2)x_2\Phi_{K^*}(x_3)X^L(x_2)+\frac{1}{2}r_2^2\Phi_{K^*}(x_3)X^t(x_2) \\
&-r_{K^*}(1-r_2^2)x_3\Phi_{K^*}^s(x_3)X^L(x_2)+r_{K^*}\left(2r_2^2x_2+(1-2r_2^2)x_3\right)\Phi_{K^*}^t(x_3)X^L(x_2)\Big] \\
&\times E_6(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\;, \qquad (86)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{N4}^{XK^*} &= 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1) \\
&\times r_2\Big\{\Big[(1-r_2^2)(1-x_2)\Psi_{K^*}^T(x_3)X^V(x_2) \\
&+\zeta_X\frac{1}{2}r_{K^*}((1+r_2^2)\Phi_{K^*}^v(x_3)-(1-r_2^2)\Psi_{K^*}^a(x_3))X^T(x_2)\Big]E_4(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1) \\
&+\Big[(1-r_2^2)x_2\Psi_{K^*}^T(x_3)X^V(x_2)
\end{aligned}
$$

$$
\begin{aligned}
& -\left((1-r_2^2)\Phi^T_{K^*}(x_3)-\frac{1}{2}r_{K^*}((1+r_2^2)\Phi^v_{K^*}(x_3)+(1-r_2^2)\Psi^a_{K^*}(x_3)\right)X^T(x_2) \\
& -2r_{K^*}((1+r_2^2)x_2+(1-r_2^2)x_3)\Phi^v_{K^*}(x_3)X^V(x_2)\Big]E_4(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\ , \quad (87)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{N6}^{XK^*} &= -\zeta_X 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1) \\
&\times r_2\Big\{\Big[(1-r_2^2)(1-x_2)\Psi^T_{K^*}(x_3)X^V(x_2) \\
&-\zeta_X\left((1-r_2^2)\Phi^T_{K^*}(x_3)-\frac{1}{2}r_{K^*}((1+r_2^2)\Phi^v_{K^*}(x_3)+(1-r_2^2)\Psi^a_{K^*}(x_3)\right)X^T(x_2) \\
&-2r_{K^*}((1+r_2^2)(1-x_2)+(1-r_2^2)x_3)\Phi^v_{K^*}(x_3)X^V(x_2)\Big]E_6(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1) \\
&+\Big[(1-r_2^2)x_2\Psi^T_{K^*}(x_3)X^V(x_2) \\
&+\frac{1}{2}r_{K^*}((1+r_2^2)\Phi^v_{K^*}(x_3)-(1-r_2^2)\Psi^a_{K^*}(x_3))X^T(x_2)\Big]E_6(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}, \quad (88)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{T4}^{XK^*} &= 32\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1) \\
&\times r_2\Big\{\Big[(1-x_2)\Phi^T_{K^*}(x_3)X^V(x_2)-\zeta_X\frac{1}{2}r_{K^*}\left(\Phi^v_{K^*}(x_3)-(1+2r_2^2)\Psi^a_{K^*}(x_3)\right)X^T(x_2)\Big] \\
&\times E_4(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1) \\
&+\Big[x_2\Phi^T_{K^*}(x_3)X^V(x_2)-\left(\Phi^T_{K^*}(x_3)-\frac{1}{2}r_{K^*}(\Phi^v_{K^*}(x_3)+(1+2r_2^2)\Psi^a_{K^*}(x_3))\right)X^T(x_2) \\
&-2r_{K^*}((1+2r_2^2)x_2+x_3)\Phi^a_{K^*}(x_3)X^V(x_2)\Big]E_4(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\ , \qquad (89)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{T6}^{XK^*} &= -\zeta_X 32\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1) \\
&\times r_2\Big\{\Big[(1-x_2)\Phi^T_{K^*}(x_3)X^V(x_2) \\
&-\zeta_X\left(\Phi^T_{K^*}(x_3)-\frac{1}{2}r_{K^*}(\Phi^v_{K^*}(x_3)+(1+2r_2^2)\Psi^a_{K^*}(x_3))\right)X^T(x_2) \\
&-2r_{K^*}((1+2r_2^2)(1-x_2)+x_3)\Phi^a_{K^*}(x_3)X^V(x_2)\Big]E_6(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1) \\
&+\Big[x_2\Phi^T_{K^*}(x_3)X^V(x_2)-\frac{1}{2}r_{K^*}\left(\Phi^v_{K^*}(x_3)-(1+2r_2^2)\Psi^a_{K^*}(x_3)\right)X^T(x_2)\Big] \\
&\times E_6(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\ , \qquad (90)
\end{aligned}
$$

with $\zeta_{J/\psi}=+1$ and $\zeta_{\chi_{c1}}=-1$ in all the above factorization formulas.

A.2 $B\to(\chi_{c0},\eta_c)K^{(*)}$

We present the spectator amplitudes for the $B\to(\chi_{c0},\eta_c)K^{(*)}$ decays below, where the symbol X represents the charmonium χ_{c0} or η_c:

$$
\mathcal{M}_4^{XK} = 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1)
$$

$$
\begin{aligned}
&\times\Big\{\Big[(1-x_2)\Phi_K(x_3)X^v(x_2)-\zeta_X r_2^2(2r_K\Phi_K^p(x_3)-\frac{1}{2}\Phi_K(x_3))X^s(x_2)\\
&+r_K(1-r_2^2)x_3\Phi_K^\sigma(x_3)X^v(x_2)-r_K\left(2r_2^2(1-x_2)+(1-r_2^2)x_3\right)\Phi_K^p(x_3)X^v(x_2)\Big]\\
&\times E_4(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1)\\
&-\Big[(x_2+(1-2r_2^2)x_3)\Phi_K(x_3)X^v(x_2)+r_2^2(2r_K\Phi_K^p(x_3)-\frac{1}{2}\Phi_K(x_3))X^s(x_2)\\
&-r_K(1-r_2^2)x_3\Phi_K^\sigma(x_3)X^v(x_2)-r_K\left(2r_2^2x_2+(1-r_2^2)x_3\right)\Phi_K^p(x_3)X^v(x_2)\Big]\\
&\times E_4(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\ , \qquad (91)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_6^{XK} &= \zeta_X 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1)\\
&\times\Big\{\Big[(1-x_2+(1-2r_2^2)x_3)\Phi_K(x_3)X^v(x_2)-\zeta_X r_2^2(2r_K\Phi_K^p(x_3)-\frac{1}{2}\Phi_K(x_3))X^s(x_2)\\
&-r_K(1-r_2^2)x_3\Phi_K^\sigma(x_3)X^v(x_2)-r_K\left(2r_2^2(1-x_2)+(1-r_2^2)x_3\right)\Phi_K^p(x_3)X^v(x_2)\Big]\\
&\times E_6(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1)\\
&-\Big[x_2\Phi_K(x_3)X^v(x_2)+r_2^2(2r_K\Phi_K^p(x_3)-\frac{1}{2}\Phi_K(x_3))X^s(x_2)\\
&+r_K(1-r_2^2)x_3\Phi_K^\sigma(x_3)X^v(x_2)-r_K\left(2r_2^2x_2+(1-r_2^2)x_3\right)\Phi_K^p(x_3)X^v(x_2)\Big]\\
&\times E_6(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\ , \qquad (92)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_4^{XK^*} &= 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1)\\
&\times\Big\{\Big[(1-x_2)\Phi_{K^*}(x_3)X^v(x_2)-\zeta_X r_2^2(2r_{K^*}\Phi_{K^*}^s(x_3)-\frac{1}{2}\Phi_{K^*}(x_3))X^s(x_2)\\
&+r_{K^*}(1-r_2^2)x_3\Phi_{K^*}^t(x_3)X^v(x_2)-r_{K^*}\left(2r_2^2(1-x_2)+(1-r_2^2)x_3\right)\Phi_{K^*}^s(x_3)X^v(x_2)\Big]\\
&\times E_4(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1)\\
&-\Big[(x_2+(1-2r_2^2)x_3)\Phi_{K^*}(x_3)X^v(x_2)+r_2^2(2r_{K^*}\Phi_K^s(x_3)-\frac{1}{2}\Phi_{K^*}(x_3))X^s(x_2)\\
&-r_{K^*}(1-r_2^2)x_3\Phi_{K^*}^t(x_3)X^v(x_2)-r_{K^*}\left(2r_2^2x_2+(1-r_2^2)x_3\right)\Phi_{K^*}^s(x_3)X^v(x_2)\Big]\\
&\times E_4(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\ , \qquad (93)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_6^{XK^*} &= \zeta_X 16\pi C_F\sqrt{2N_c}\int_0^1[dx]\int_0^\infty b_1db_1\Phi_B(x_1,b_1)\\
&\times\Big\{\Big[(1-x_2+(1-2r_2^2)x_3)\Phi_{K^*}(x_3)X^v(x_2)-\zeta_X r_2^2(2r_K\Phi_{K^*}^s(x_3)-\frac{1}{2}\Phi_{K^*}(x_3))X^s(x_2)\\
&-r_{K^*}(1-r_2^2)x_3\Phi_{K^*}^t(x_3)X^v(x_2)-r_{K^*}\left(2r_2^2(1-x_2)+(1-r_2^2)x_3\right)\Phi_{K^*}^s(x_3)X^v(x_2)\Big]\\
&\times E_6(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1)
\end{aligned}
$$

$$
\begin{aligned}
&-\Big[x_2\Phi_{K^*}(x_3)X^v(x_2)+r_2^2(2r_{K^*}\Phi^s_{K^*}(x_3)-\frac{1}{2}\Phi_{K^*}(x_3))X^s(x_2)\\
&+r_{K^*}(1-r_2^2)x_3\Phi^t_{K^*}(x_3)X^v(x_2)-r_{K^*}\left(2r_2^2x_2+(1-r_2^2)x_3\right)\Phi^s_{K^*}(x_3)X^v(x_2)\Big]\\
&\times E_6(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1)\Big\}\ ,
\end{aligned}
\tag{94}
$$

with $\zeta_{\chi_{c0}}=+1$ and $\zeta_{\eta_c}=-1$.

References

[1] M. Bauer, B. Stech, M. Wirbel, Z. Phys. C **29**, 637 (1985); *ibid.* **34**, 103 (1987).

[2] A. Ali, G. Kramer and C.D. Lü, Phys. Rev. D **58**, 094009 (1998); Y.H. Chen, H.Y. Cheng, B. Tseng, and K.C. Yang, Phys. Rev. D **60**, 094014 (1999).

[3] H.Y. Cheng and K.C. Yang, Phys. Rev. D **59**, 092004 (1999).

[4] M. Neubert and A.A. Petrov, Phys. Lett. B **519**, 50 (2001).

[5] BABAR Collaboration, B. Aubert *et al.*, hep-ex/0412062.

[6] H-n. Li, Prog. Part. Nucl. Phys. **51**, 85 (2003); Czech. J. Phys. **53**, 657 (2003).

[7] J. Chay and C. Kim, hep-ph/0009244.

[8] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000).

[9] H.Y. Cheng and K.C. Yang, Phys. Rev. D **63**, 074011 (2001).

[10] Z.Z. Song, C. Meng, and K.T. Chao, Phys. Lett. B **568**, 127 (2003); Z.Z. Song, C. Meng, Y.J. Gao, and K.T. Chao, Phys. Rev. D **69**, 054009 (2004).

[11] Z.Z. Song, C. Meng, and K.T. Chao, Eur. Phys. J. C **36**, 365 (2004).

[12] B. Melic, Lect. Notes Phys. **647**, 287 (2004); Phys. Rev. D **68**, 034004 (2003); B. Melic and R. Rückl, hep-ph/0210353.

[13] Z.G. Wang, L. Li, and T. Huang, Phys. Rev. D **70**, 074006 (2004).

[14] H-n. Li and H.L. Yu, Phys. Rev. Lett. **74**, 4388 (1995); Phys. Lett. B **353**, 301 (1995); Phys. Rev. D **53**, 2480 (1996).

[15] C.H. Chang and H-n. Li, Phys. Rev. D **55**, 5577 (1997).

[16] Y.Y. Keum, H-n. Li, and A.I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001); Y.Y. Keum and H-n. Li, Phys. Rev. **D63**, 074006 (2001).

[17] C. D. Lü, K. Ukai, and M. Z. Yang, Phys. Rev. D **63**, 074009 (2001).

[18] Y.Y. Keum, T. Kurimoto, H.-n. Li, C.D. Lu, and A.I. Sanda, Phys. Rev. D **69**, 094018 (2004).

[19] A.E. Bondar and V.L. Chernyak, hep-ph/0412335.

[20] T.W. Yeh and H-n. Li, Phys. Rev. D **56**, 1615 (1997).

[21] A.I. Sanda, N. Sinha, R. Sinha, and K. Ukai, in *Proceedings for the 4th International Workshop on B Physics and CP Violation*, ed. T. Ohshima and A.I. Sanda (World Scientific, Singapore, 2001).

[22] C.H. Chen, Phys. Rev. D **67**, 094011 (2003).

[23] P. Ball, JHEP **01**, 010 (1999).

[24] P. Ball, V.M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. **B529**, 323 (1998).

[25] Quarkonium Working Group, N. Brambilla *et al.*, hep-ph/0412158.

[26] M. Nagashima and H-n. Li, Phys. Rev. D **67**, 034001 (2003).

[27] C.H. Chen, Y.Y. Keum, and H-n. Li, Phys. Rev. D **64**, 112002 (2001); Phys. Rev. D **66**, 054013 (2002).

[28] T. Kurimoto, H-n. Li, and A.I. Sanda, Phys. Rev. D **65**, 014007, (2002); Phys. Rev. D **67**, 054028 (2003).

[29] H-n. Li and H.S. Liao, Phys. Rev. D **70**, 074030 (2004).

[30] J.C. Collins and D.E. Soper, Nucl. Phys. **B193**, 381 (1981).

[31] J. Botts and G. Sterman, Nucl. Phys. **B325**, 62 (1989).

[32] H-n. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).

[33] H-n. Li and S. Mishima, hep-ph/0411146, to appear in Phys. Rev. D.

[34] J.P. Ma and Z.G. Si, Phys. Rev. D **70**, 074007 (2004).

[35] Particle Data Group Collaboration, S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).

[36] D.S. Hwang and G.H. Kim, Z. Phys. C **76**, 107 (1997).

[37] V.A. Novikov, L.B. Okun, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, Phys. Rep. **41**, 1 (1978).

[38] H.Y. Cheng, C.K. Chua, and C.W. Hwang, Phys. Rev. D **69**, 074025 (2004) and references therein; H.Y. Cheng, hep-ph/0410316.

[39] B. Melic, Phys. Lett. B **591**, 91 (2004).

[40] BELLE Collaboration, K. Abe *et al.*, hep-ex/0408104.

[41] T.N. Pham and G. Zhu, hep-ph/0412428.

[42] C. Meng, Y.J. Gao, and K.T. Chao, hep-ph/0502240.

[43] BABAR Collaboration, B. Aubert *et al.*, hep-ex/0501061.

[44] BELLE Collaboration, A. Garmash *et al.*, hep-ex/0412066.

[45] BELLE Collaboration, F. Fang *et al.*, Phys. Rev. Lett. **90**, 071801 (2003).

[46] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. D **70**, 011101 (2004).